



HUNGARIAN UNIVERSITY OF  
AGRICULTURE AND LIFE SCIENCES

Development of indoor precision vegetable  
production  
system and its mathematical models

PhD Thesis

by

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## INTRODUCTION, OBJECTIVES

### 1. INTRODUCTION, OBJECTIVES

Controlled Environment Agriculture (CEA) enables precise control of environmental factors. Recent advances have enhanced environmental monitoring and resource management in indoor farming systems. However, many existing approaches still rely on static control strategies and limited integration between data-driven methods and biological growth models. This thesis proposes an integrated framework for plant growth optimization. Lettuce (*Lactuca sativa* L.) was selected as the model crop for experimental validation.

- a) To evaluate a four-dimensional logistic growth model under dynamic environmental conditions.
- b) To develop and validate a multivariate regression model relating relative growth rate, light intensity, photoperiod, and plant maturity.
- c) To compare canopy- and lamp-level Light Use Efficiency (LUEP and LUEL) as functions of Leaf Area Index (LAI) and identify the optimal canopy density.
- d) To determine machine-learned weighting coefficients for a composite Plant Growth Index (PGI) quantifying plant health and resource-use efficiency.
- e) To identify nutrient concentration thresholds beyond which additional inputs provide limited or no growth benefits.
- f) To establish and validate an allometric relationship between plant height and biomass for lettuce (*Lactuca sativa* L.).

# MATERIALS AND METHODS

## 2. MATERIALS AND METHODS

### 2.1. Experimental Setup for Plant Growth Evaluation

This study evaluated an IoT-based indoor farming system using lettuce (*Lactuca sativa* L.) as the model crop. Experiments were conducted over 8-week growth periods under controlled environmental conditions. The overall IoT architecture is presented in Fig. 2.1.

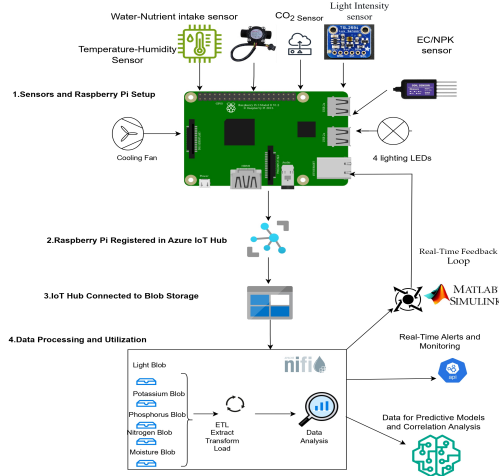


Fig. 2.1 IoT-Based pipeline for plant growth optimization

### 2.2. Mathematical Modeling of Plant Growth

#### 2.2.1. Multidimensional Logistic Growth state vector $X(t)$

To extend the traditional logistic growth model to accommodate multidimensional traits, I define a multidimensional logistic growth state vector  $X(t)$  that

## MATERIALS AND METHODS

incorporates the key plant traits as in Equation (1):

$$X(t) = \begin{bmatrix} H(t) \\ B(t) \\ C(t) \\ A(t) \end{bmatrix} \quad (1)$$

I redefine the carrying capacity  $K$  and growth rate  $r$  as vectors as represented in Equation (2) and Equation (3) respectively:

$$K = \begin{bmatrix} K_H \\ K_B \\ K_C \\ K_A \end{bmatrix} \quad (2), \quad r = \begin{bmatrix} r_H \\ r_B \\ r_C \\ r_A \end{bmatrix} \quad (3)$$

Therefore, the logistic growth equation becomes as in Equation (4):

$$\frac{dX(t)}{dt} = r \odot \left(1 - \frac{X}{K}\right) \odot X \quad (4)$$

where  $\odot$  denotes element-wise multiplication,  $r$  is the vector of growth rates,  $K$  = the vector of carrying capacities, and division of vectors is performed element-wise.

where:

- $r = [r_H, r_B, r_C, r_A]^T$  is the vector of growth rates,
- $K = [K_H, K_B, K_C, K_A]^T$  is the vector of carrying capacities,
- $\odot$  denotes element-wise multiplication,
- The division  $\frac{X}{K}$  is also performed element-wise.

## MATERIALS AND METHODS

### 2.2.2. Proposed Interdependence Modeling in the 4D Model

In the extended four-dimensional logistic growth model, the plant traits biomass (B), leaf area (A), chlorophyll content (C), and plant height (H) were treated as interdependent variables that dynamically influence one another.

I hypothesized that  $g(B(t))$  follows an allometric relationship of the form shown in Equation (5):

$$H(t) = \alpha_H \cdot B(t)^{\beta_H} \quad (5)$$

where  $\alpha_H$  and  $\beta_H$  are species-specific parameters. I tested this hypothesis empirically using the collected dataset.

### 2.3. Machine Learning and Data Processing Analytics

Fig. 2.2 shows the prototype growing chamber.

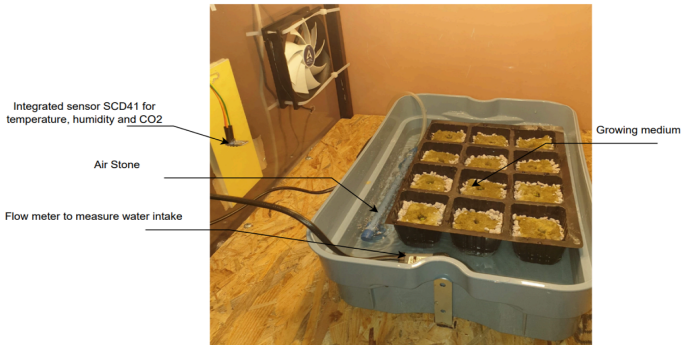


Fig. 2.2. IoT-Integrated Growing Chamber from inside

## MATERIALS AND METHODS

Fig. 2.3 shows the prototype electronics.

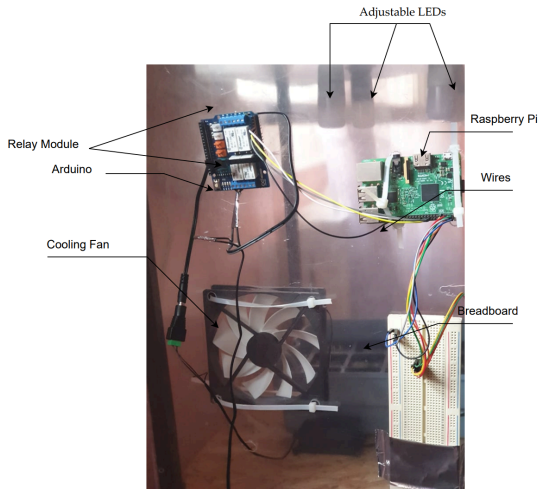


Fig. 2.3. Side view of the prototype.

Seedlings were monitored inside the controlled chamber as shown in Fig. 2.4.

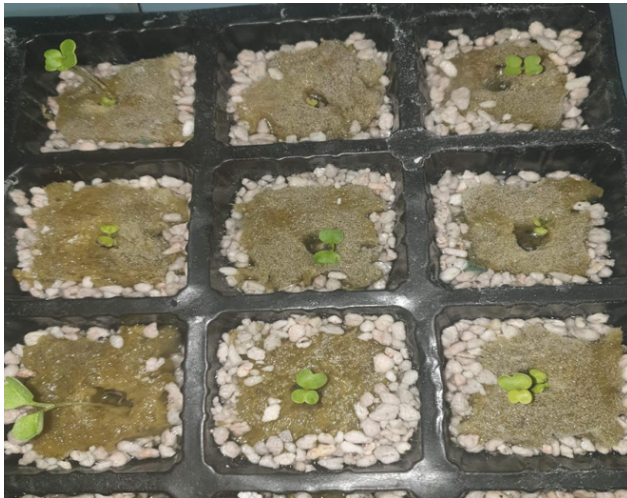


Fig. 2.4. Early growth stage of lettuce (*Lactuca sativa* L.).

## MATERIALS AND METHODS

### 2.4. Novel Growth Performance Metrics

#### 2.4.1. The Growth Efficiency ratio GER

The Growth Efficiency Ratio (GER), defined in Equation (6), was developed to evaluate resource-use efficiency in biomass production, a key factor in energy-intensive indoor farming systems.

$$\text{GER} = \frac{B^*(t)}{\text{CRU}(t)} = \frac{B^*(t)}{E^*(t) + W^*(t) + N^*(t)} \quad (6)$$

The Combined Resource Use (CRU) represents the aggregated normalized resource input in dimensionless form and serves as the denominator of the GER metric, as defined in Equation (7).

$$\text{CRU}(t) = E^*(t) + W^*(t) + N^*(t) \quad (7)$$

$$\text{Where: } B^*(t) = \frac{B(t)}{B_{\max}}, \quad W^*(t) = \frac{W(t)}{W_{\max}}, \quad N^*(t) = \frac{N(t)}{N_{\max}}, \\ E^*(t) = \frac{E(t)}{E_{\max}}$$

where  $B(t)$ ,  $E(t)$ ,  $W(t)$ , and  $N(t)$  denote biomass, energy, water, and nutrient inputs at time  $t$ . Their normalized forms are represented by  $B^*$ ,  $E^*$ ,  $W^*$ , and  $N^*$ , using the corresponding maximum values  $B_{\max}$ ,  $E_{\max}$ ,  $W_{\max}$ , and  $N_{\max}$ .

#### 2.4.2. Plant Growth Index (PGI)

The Plant Growth Index (PGI), developed in this study, integrates plant height, biomass, and leaf area into a single metric of plant health and resource efficiency as shown in Equation (8).

## MATERIALS AND METHODS

$$PGI = w_1 \cdot \frac{H}{H_{\max}} + w_2 \cdot \frac{B}{B_{\max}} + w_3 \cdot \frac{A}{A_{\max}} \quad (8)$$

$$w_1 = \frac{H_{\text{norm}}}{H_{\text{norm}} + B_{\text{norm}} + A_{\text{norm}}}, w_2 = \frac{B_{\text{norm}}}{H_{\text{norm}} + B_{\text{norm}} + A_{\text{norm}}}, w_3 = \frac{A_{\text{norm}}}{H_{\text{norm}} + B_{\text{norm}} + A_{\text{norm}}}$$

Where  $X_{\text{norm}} = \frac{\sum_{i=1}^n X_i}{X_{\max}}$  and  $X \in \{H, B, A\}$ ,  $i$  represents the day of measurement, and  $n$  denotes the total number of measured days.  $H$  represents plant height normalized by the maximum observed height,  $B$  represents plant biomass normalized by the maximum observed biomass, and  $A$  represents leaf area normalized by the maximum observed leaf area.

### 2.4.3. Relative Growth Rate (RGR) Metric

The Relative Growth Rate is defined in Equation (9):

$$RGR = \frac{B_{t+1} - B_t}{B_t} \quad (9)$$

where:

$B_t$  is plant biomass at week  $t$ . [g],  $B_{t+1}$  is plant biomass at week  $t + 1$ . [g]

Using the centered variables, the growth response was approximated by the following second-order regression model in Equation (10):

$$RGR = \beta_0 + \beta_1 I_c + \beta_2 P_c + \beta_3 W_c + \beta_{11} I_c^2 + \beta_{22} P_c^2 + \beta_{33} W_c^2 + \beta_{12} I_c P_c + \beta_{13} I_c W_c + \beta_{23} P_c W_c \quad (10)$$

## MATERIALS AND METHODS

To improve numerical stability during regression estimation, the predictors were centered around their mean values:

$$I_c = I - \bar{I}, P_c = P - \bar{P}, W_c = W - \bar{W}$$

Where:  $I$  represents light intensity ( $\mu\text{mol m}^{-2} \text{s}^{-1}$ ),  $P$  represents photoperiod ( $\text{hours day}^{-1}$ ),  $W$  represents cultivation week, and  $\bar{I}$ ,  $\bar{P}$ , and  $\bar{W}$  denote the mean values of light intensity, photoperiod, and cultivation week, respectively.

The regression coefficients therefore have the following units:  $\beta_0$  is dimensionless,  $\beta_1 (\mu\text{mol m}^{-2} \text{s}^{-1})^{-1}$ ,  $\beta_2 (\text{h day}^{-1})^{-1}$ ,  $\beta_3$  dimensionless,  $\beta_{12} (\mu\text{mol m}^{-2} \text{s}^{-1} \cdot \text{h day}^{-1})^{-1}$ ,  $\beta_{23} (\text{h day}^{-1})^{-1}$ ,  $\beta_{22} (\text{h day}^{-1})^{-2}$ , and  $\beta_{33}$  dimensionless.

The final model retained only the statistically relevant predictors shown in Equation (11):

$$\text{RGR} = \beta_0 + \beta_1 I_c + \beta_2 P_c + \beta_3 W_c + \beta_{12} I_c P_c + \beta_{23} P_c W_c + \beta_{22} P_c^2 + \beta_{33} W_c^2 \quad (11)$$

The terms  $I_c^2$  and  $I_c W_c$  were removed during stepwise selection because they did not significantly improve model performance. The regression coefficients were estimated using ordinary least squares (OLS) implemented in MATLAB.

## MATERIALS AND METHODS

The model can be written in matrix form as in Equation (12):

$$y = X\beta + \varepsilon \quad (12)$$

where:  $y$  represents the vector of observed RGR values,  $X$  is the regression design matrix,  $\beta$  is the vector of regression coefficients,  $\varepsilon$  is the residual error vector.

The design matrix was constructed as Equation (13):

$$X = [1 \ I_c \ P_c \ W_c \ I_c \cdot P_c \ P_c \cdot W_c \ P_c.^2 \ W_c.^2] \quad (13)$$

The OLS estimator is shown in Equation (14):

$$\beta = (X^T X)^{-1} X^T y \quad (14)$$

### 2.4.4. Light Use Efficiency (LUE) Metrics

LUEP and LUEL are key metrics for assessing the efficiency of artificial lighting in biomass production. LUEP is calculated using Equation (15):

$$LUEP = \frac{\text{PAR Absorbed by Leaves}}{\text{PAR emitted by Lamps}} \quad (15)$$

LUEL metric represents the efficiency of converting light emitted from lamps into usable energy for plants. It can be computed according to the Equation (16):

$$LUEL = \frac{\text{PAR Received at the Canopy}}{\text{PAR emitted by Lamps}} \quad (16)$$

## MATERIALS AND METHODS

### 2.5. Image Analysis

Plant images in Fig. 2.5 (week 5) and Fig. 2.6 (week 8) were captured under controlled lighting conditions and processed using PlantCV and OpenCV.

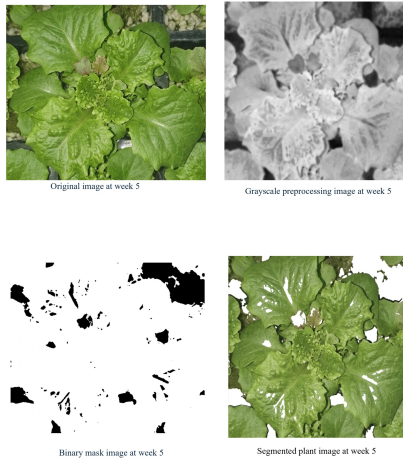


Fig. 2.5. Plant image segmentation workflow at week 5.

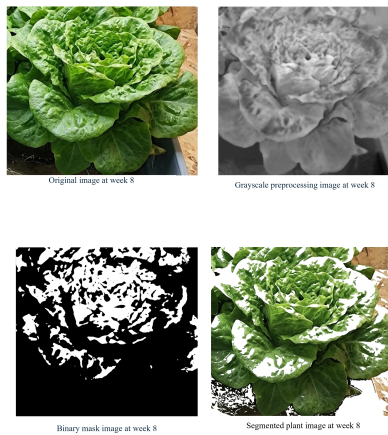


Fig. 2.6. Plant image segmentation workflow at week 8.

### 3. RESULTS AND DISCUSSION

#### 3.1. Growth Efficiency Ratio GER

The plot in Fig. 4.1 shows X-axis representing energy consumption (kWh used per plant or system) and Y-axis representing the Growth Efficiency Ratio (GER).

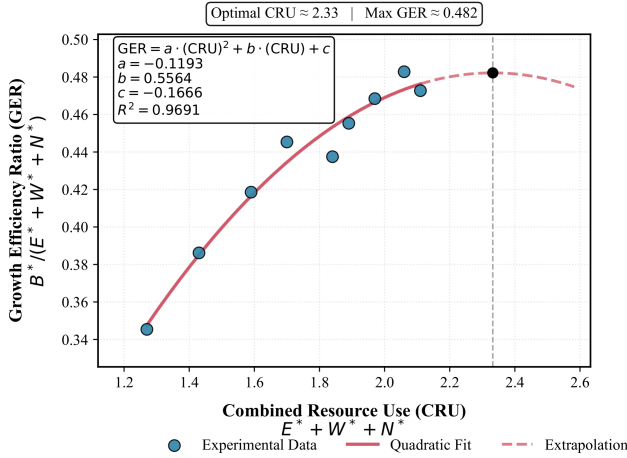


Fig. 3.1. Growth Efficiency Ratio vs Combined Resource Use Quadratic Regression Analysis

Fig. 3.1 shows that GER increased with CRU but exhibited diminishing returns at higher input levels. The relationship was described by the following quadratic model:

$$\text{GER} = a(\text{CRU})^2 + b(\text{CRU}) + c$$

where CRU represents the combined normalized energy, water, and nutrient inputs. The fitted model ( $R^2 = 0.9691$ ) confirmed an optimal resource-use region for growth efficiency. The regression coefficients were:

$$a = -0.1193, b = 0.5564, c = -0.1666$$

## RESULTS AND DISCUSSION

### 3.2. Machine Learning for Plant Growth Index (PGI)

A linear regression model applied to normalized growth data yielded PGI weights of  $W_1 = 0.3335$  (height),  $W_2 = 0.3264$  (biomass), and  $W_3 = 0.3401$  (leaf area). As shown in Fig. 3.2, PGI exhibited a sigmoid growth pattern over 30 days, confirming its suitability as a composite indicator of plant health and growth efficiency.

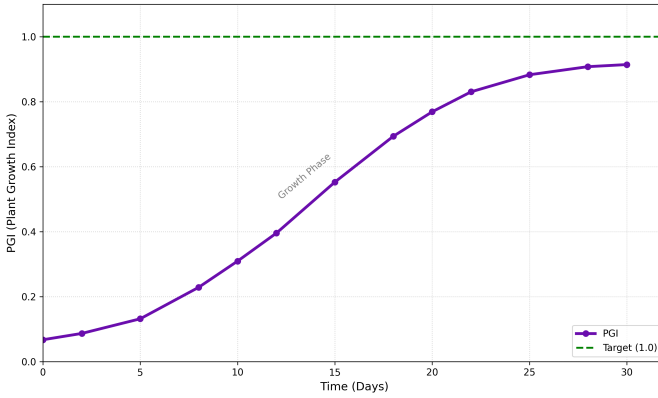


Fig. 3.2. PGI (Plant Growth Index) over 30 Days.

### 3.3. Multivariate Regression Analysis of Growth Drivers

To quantify the effects of environmental variables on plant growth, a multivariate linear regression model was applied using observed growth as the dependent variable. The results identified phosphorus as the strongest positive contributor ( $\beta = 0.54$ ), followed by temperature ( $\beta = 0.23$ ), while potassium showed a strong negative effect ( $\beta = -0.43$ ). Moisture ( $\beta = 0.06$ ), nitrogen ( $\beta = 0.04$ ), and light intensity ( $\beta = 0.03$ ) exhibited relatively weak contributions.

## RESULTS AND DISCUSSION

### 3.4. IoT Framework Performance

#### 3.4.1. Analysis of Light intensity

Fig. 3.3 presents a three-dimensional RSM plot of Relative Growth Rate (RGR) as a function of light intensity and time, revealing a nonlinear relationship and the combined effects of light intensity and plant maturity on growth.

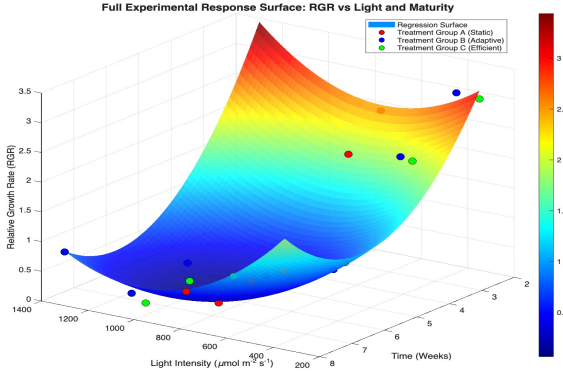


Fig. 3.3. Growth rate vs Light Intensity and Time.

#### 3.4.2. Development of the Growth Response Model

Applying the response surface regression model to the experimental dataset produced the coefficient estimates in Equation (17):

$$\begin{aligned} \text{RGR} = & 0.4253 - 4.09 \times 10^{-4}I_c - 0.0708P_c - \\ & 0.3074W_c + 0.00334(I_cP_c) - 0.3351(P_cW_c) - \\ & 0.1331P_c^2 + 0.1250W_c^2 \end{aligned} \quad (17)$$

## RESULTS AND DISCUSSION

### 3.4.3. RGR Model Validation

Fig. 3.4 compares experimentally measured and predicted Relative Growth Rate (RGR) values for Treatment Groups A, B, and C over weeks 2–8, demonstrating the predictive performance of the stepwise regression model.

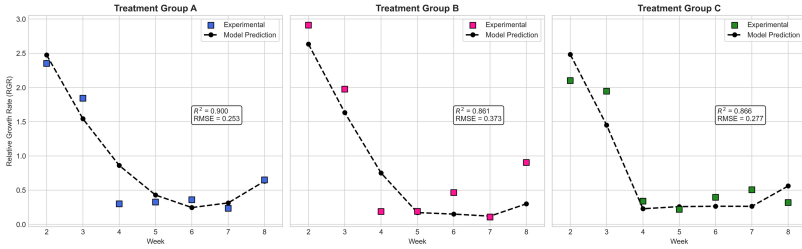


Fig. 3.4. Predictive Accuracy of the RGR Model Across Treatment Groups A, B and C

Model performance was evaluated using  $R^2$  and RMSE. The results for each treatment group are summarized below:

Treatment Group A:  $R^2 = [0.900]$ , RMSE = [0.253]

Treatment Group B:  $R^2 = [0.920]$ , RMSE = [0.283]

Treatment Group C:  $R^2 = [0.866]$ , RMSE = [0.277]

### 3.5. Energy Efficiency Analysis

Fig. 3.5 illustrates the relationship between Leaf Area Index (LAI) and light use efficiency metrics (LUEP and LUEL). LUEP and LUEL were computed from the four configurations.

## RESULTS AND DISCUSSION

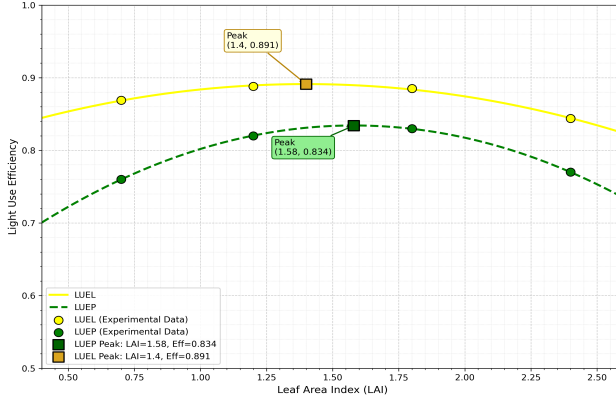


Fig.3.5. Light Use Efficiency vs. LAI.

The equations were fitted using a polynomial regression approach, leading to the following results in Equation (18) and Equation (19):

$$\text{LUEP} = 0.5948 + 0.3029x - 0.0958x^2 \quad (18)$$

$$\text{LUEL} = 0.7999 + 0.1306x - 0.0467x^2 \quad (19)$$

### 3.6. Allometric Relationship Between Height (H) and Biomass (B)

Fig. 3.6 shows height–biomass scaling ( $H = \alpha B^\beta$ ).

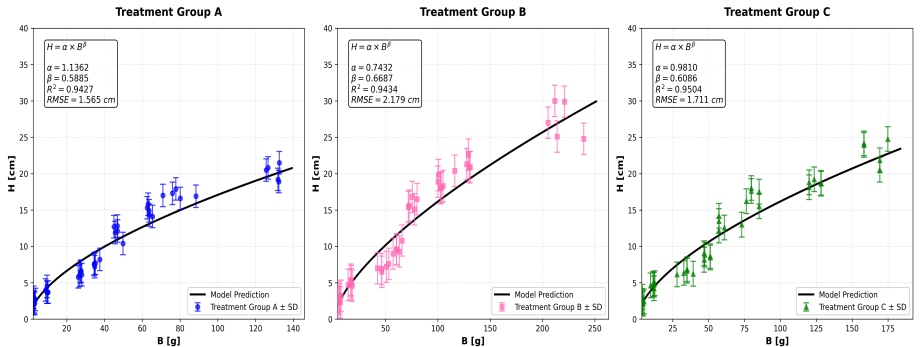


Fig. 3.6. Allometric Relationship Between Height (H) and Biomass (B).

## RESULTS AND DISCUSSION

Fig. 3.7 shows strong allometric relationships between plant height and biomass across all treatment groups ( $R^2 = 0.9427$ – $0.9504$ ), with Treatment Group B exhibiting the highest scaling exponent ( $\beta = 0.6687$ ). A global scaling function was then fitted using pooled data from all groups.

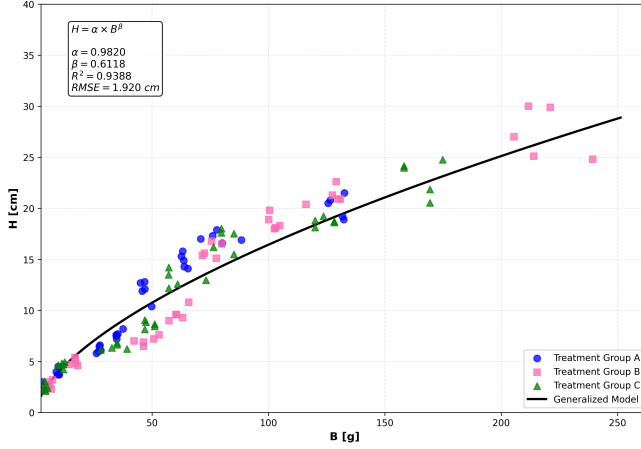


Fig. 3.7. Generalized Allometric Model  $H = \alpha B^\beta$

### 3.7. Advanced Growth Four-Dimensional Model Validation

The carrying capacity ( $K$ ) and intrinsic growth rate ( $r$ ) were estimated from the observed data. Equations (20)–(23) present the calibrated parameters for Treatment Group A, and Fig. 3.8 shows the fitted logistic curves against the measured growth traits.

$$S_A = \begin{cases} \text{Chlorophyll:} & K = 98.38, r = 0.166 \\ \text{Biomass:} & K = 168.85, r = 0.716 \\ \text{Height:} & K = 25.2, r = 0.509 \\ \text{Leaf Area:} & K = 214.05, r = 0.581 \end{cases} \quad (20-23)$$

## RESULTS AND DISCUSSION

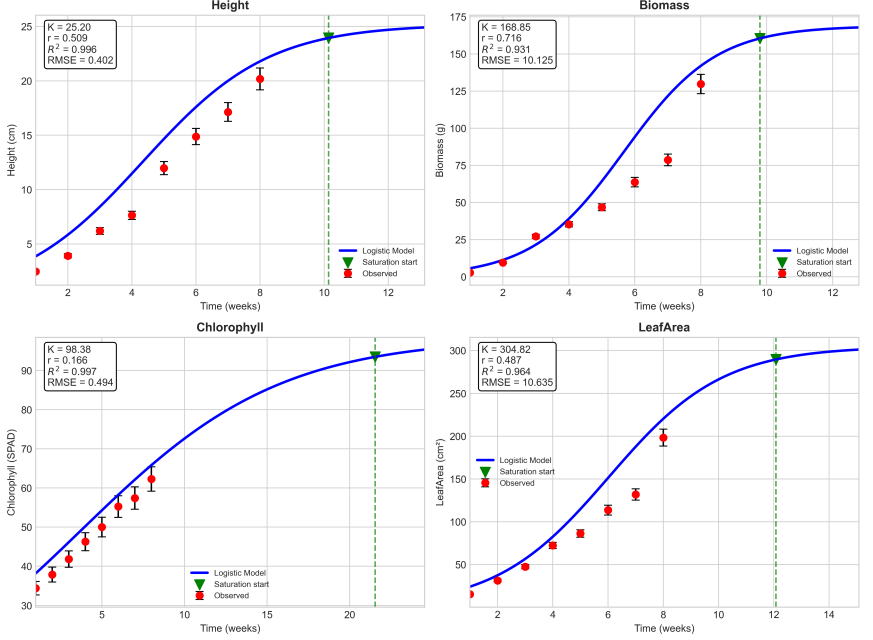


Fig. 3.8. Measured Growth Traits vs Logistic Model (Treatment Group A)

The final calibrated parameters for Treatment Group B are summarized below in Equation (24), Equation (25), Equation (26) and Equation (27). The corresponding fitted logistic curves and their agreement with the measured growth traits are illustrated in Fig. 3.9 (Measured Growth Traits vs Logistic Model Treatment Group B):

$$S_B = \begin{cases} \text{Chlorophyll:} & K = 70.68 \text{ SPAD, } r = 0.274 \text{ week}^{-1} \\ \text{Biomass:} & K = 332.96 \text{ g, } r = 0.688 \text{ week}^{-1} \\ \text{Height:} & K = 37.20 \text{ cm, } r = 0.546 \text{ week}^{-1} \\ \text{Leaf Area:} & K = 292.09 \text{ cm}^2, r = 0.594 \text{ week}^{-1} \end{cases}$$

(24-27)

## RESULTS AND DISCUSSION

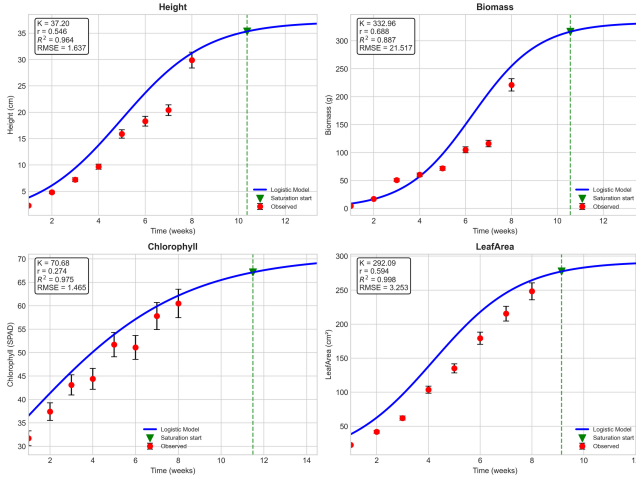


Fig. 3.9. Measured Growth Traits vs Logistic Model (Treatment Group B)

Equations (28)–(31) present the calibrated parameters for Treatment Group C, while Fig. 3.10 compares the fitted logistic curves with the measured growth traits.

$$S_C = \begin{cases} \text{Chlorophyll:} & K = 67.55 \text{ SPAD}, r = 0.326 \text{ week}^{-1} \\ \text{Biomass:} & K = 196.21 \text{ g}, r = 0.743 \text{ week}^{-1} \\ \text{Height:} & K = 32.76 \text{ cm}, r = 0.459 \text{ week}^{-1} \\ \text{Leaf Area:} & K = 214.19 \text{ cm}^2, r = 0.806 \text{ week}^{-1} \end{cases} \quad (28-31)$$

A generalized logistic model was developed by pooling data from all treatment groups as shown in Fig. 3.11.

## RESULTS AND DISCUSSION

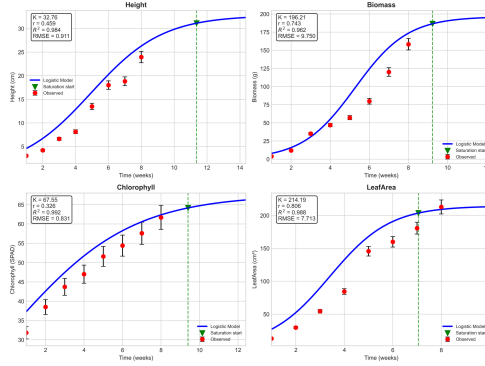


Fig. 3.10. Measured Growth Traits vs Logistic Model (Treatment Group C)

The estimated parameters for each trait are given in Equations (32) through (35):

$$S = \begin{cases} \text{Chlorophyll:} & K = 74.00 \text{ SPAD}, r = 0.253 \text{ week}^{-1} \\ \text{Biomass:} & K = 223.00 \text{ g}, r = 0.717 \text{ week}^{-1} \\ \text{Height:} & K = 31.60 \text{ cm}, r = 0.503 \text{ week}^{-1} \\ \text{Leaf Area:} & K = 235.90 \text{ cm}^2, r = 0.654 \text{ week}^{-1} \end{cases} \quad (32-35)$$

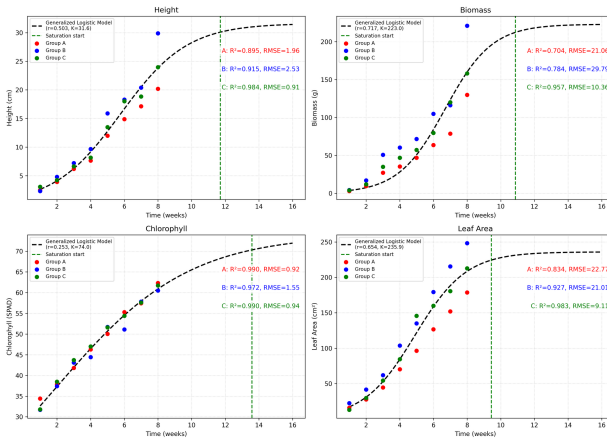


Fig. 3.11. Generalized Logistic Model Across All Treatment Groups

## NEW SCIENTIFIC RESULTS

### 4. NEW SCIENTIFIC RESULTS

#### 4.1. Multi-Dimensional Logistic Growth Model

I calibrated the model for the three experimental Treatment Groups (A, B, C) and derived a generalized Multi-Dimensional Logistic Growth Model model.

Light: 180–1300  $\mu\text{mol m}^{-2}\text{s}^{-1}$ , Temperature: 22 °C ( $\pm 1$  °C) ,  
CO<sub>2</sub>: 405–950 ppm, Nutrients: NPK 18 – 130 ppm

$$X = \begin{bmatrix} H \\ B \\ C \\ A \end{bmatrix} \text{ (Height, Biomass, Chlorophyll, Leaf Area)}$$

The Final Generalized Model in Equation (36):

$$r = \begin{bmatrix} 0.503 \\ 0.717 \\ 0.253 \\ 0.654 \end{bmatrix}, K = \begin{bmatrix} 31.6 \\ 223 \\ 74 \\ 235.9 \end{bmatrix}$$

$$\frac{dX(t)}{dt} = \begin{bmatrix} 0.503 \\ 0.717 \\ 0.253 \\ 0.654 \end{bmatrix} \odot \left( 1 - \frac{\begin{bmatrix} H \\ B \\ C \\ A \end{bmatrix}}{\begin{bmatrix} 31.6 \\ 223 \\ 74 \\ 235.9 \end{bmatrix}} \right) \odot \begin{bmatrix} H \\ B \\ C \\ A \end{bmatrix} \quad (36)$$

|             |                                                |
|-------------|------------------------------------------------|
| Height      | : $R^2 = 0.859 - 0.984$ , RMSE = 0.91 – 2.53   |
| Biomass     | : $R^2 = 0.704 - 0.957$ , RMSE = 10.36 – 29.79 |
| Chlorophyll | : $R^2 = 0.972 - 0.990$ , RMSE = 0.92 – 1.55   |
| Leaf Area   | : $R^2 = 0.834 - 0.983$ , RMSE = 9.11 – 22.77  |

## NEW SCIENTIFIC RESULTS

### 4.2.Allometric Relationship Between Height (H) and Biomass (B)

I identified the allometric relationships between plant height and biomass for Treatment Groups A, B, and C and derived a generalized model, as presented in Equations (37)–(40):

$$\begin{aligned} H &= 1.13B^{0.58} & (R^2 = 0.9427, \text{RMSE} = 1.565) \\ H &= 0.743B^{0.668} & (R^2 = 0.9434, \text{RMSE} = 2.179) \\ H &= 0.981B^{0.6086} & (R^2 = 0.9504, \text{RMSE} = 1.711) \\ H &= 0.982B^{0.6118} & (R^2 = 0.9388, \text{RMSE} = 1.920)(37-40) \end{aligned}$$

### 4.3.Relative Growth Rate Under Controlled Environmental Conditions

I derived the model in Equation (41) using stepwise regression to retain only significant predictors:

$$\begin{aligned} \text{RGR} = & 0.4253 - 4.09 \times 10^{-4}I_c - 0.0708P_c - 0.3074W_c + \\ & 0.00334(I_cP_c) - 0.3351(P_cW_c) - 0.1331P_c^2 + 0.1250W_c^2 \end{aligned} \quad (41)$$

where  $I_c$ ,  $P_c$ , and  $W_c$  are centered values of light intensity ( $\mu\text{mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$ ), photoperiod ( $\text{h} \cdot \text{day}^{-1}$ ), and time (weeks).

$$R^2 = 0.866-0.920, \text{RMSE} = 0.253-0.283$$

The model is valid over the experimental ranges:

$$\begin{aligned} I \in [180,1300] \mu\text{mol} \cdot \text{m}^{-2} \cdot \text{s}^{-1}, P \in [12,20] \text{ h} \cdot \text{day}^{-1}, \\ \text{Time (weeks)} : 2-8 \text{ weeks.} \end{aligned}$$

## NEW SCIENTIFIC RESULTS

### 4.4.Quantitative Models for Light Use Efficiency vs. Leaf Area Index (LAI)

I derived polynomial relationships between Leaf Area Index (x = LAI) and light efficiency metrics:

#### 4.4.1. Light Use Efficiency at Canopy Level (LUEP)

Equation (42) presents the LUEP model:

$$\text{LUEP} = 0.5948 + 0.3029x - 0.0958x^2 \quad (42)$$

Validity Domain:  $x \in [0.5, 2.5]$  where x is LAI.

Peak efficiency:  $\text{LUEP} = 0.834$  at  $x = 1.58$ ,  $R^2 = 0.977$ ,  
 $\text{RMSE} = 0.0046$

#### 4.4.2. Light Use Efficiency at Lamp Level (LUEL)

Equation (43) gives the lamp-level efficiency model:

$$\text{LUEL} = 0.7999 + 0.1306x - 0.0467x^2 \quad (43)$$

Validity Domain:  $x \in [0.5, 2.5]$  where x is LAI.

Peak efficiency:  $\text{LUEL} = 0.891$  at  $x = 1.40$ ,  $R^2 = 0.980$ ,  
 $\text{RMSE} = 0.0024$

### 4.5.Plant Growth Index (PGI) with Machine-Learned Weights

## NEW SCIENTIFIC RESULTS

I developed a composite Plant Growth Index. Using data-driven regression, I empirically determined the relative contribution of each trait to overall growth in Equation (44):

$$PGI = 0.3335 \cdot \frac{H}{H_{\max}} + 0.3264 \cdot \frac{B}{B_{\max}} + 0.3401 \cdot \frac{A}{A_{\max}} \quad (44)$$

Where: H = Height (cm), B = Biomass (g), A = Leaf area (cm<sup>2</sup>)

Weights (0.3335, 0.3264, 0.3401) derived from Data-Driven Proportional

$$H_{\max} = 34.09 \text{ cm}, B_{\max} = 286.87 \text{ g}, \\ A_{\max} = 304.27 \text{ cm}^2$$

### 4.6. Mathematical Model and Control Logic for Nutrient Optimization

I identified phosphorus concentration as the primary driver of Lettuce (*Lactuca sativa* L.) growth via multivariate regression shown in Equation (45).

$$\text{Growth (g/day)} = 0.54 \cdot P(t) + 0.23 \cdot T(t) - 0.43 \cdot K(t) + \epsilon > 0 \quad (45)$$

$$0.54P(t) + 0.23T(t) + \epsilon > 0.43K(t)$$

Where P(t), T(t), and K(t) represent phosphorus, temperature, and potassium, respectively;  $\epsilon$  is the error term ( $\approx \pm 0.04$  g/day).

Valid ranges: P = 20–33 mg/kg, T = 15–33 °C, K = 40–49 mg/kg; data source: Azure IoT sensors ( $\pm 5\%$  accuracy).

## 5. CONCLUSION AND SUGGESTIONS

The main objective of this research was achieved through the development of a unified IoT-enabled framework for optimizing plant growth and resource efficiency in Controlled Environment Agriculture. A four-dimensional logistic growth model was developed and validated, accurately describing biomass, height, chlorophyll content, and leaf area dynamics under different environmental conditions.

The proposed models demonstrated strong predictive performance ( $R^2 > 0.90$ ), while the generalized growth model achieved a global fit of approximately  $R^2 \approx 0.94$ . Novel performance metrics, including the Growth Efficiency Ratio (GER), Plant Growth Index (PGI), and Integrated Agricultural Efficiency Metric (IAEM), were introduced to evaluate growth performance and resource utilization.

The integration of IoT-based monitoring and adaptive control significantly improved plant growth, increasing biomass, plant height, and leaf area while enhancing resource-use efficiency. System latency was reduced from 120 ms to 85 ms ( $\approx 29\%$  improvement), ensuring reliable real-time operation.

Overall, the results demonstrate that combining IoT technologies, mathematical modeling, and machine learning provides an effective and scalable framework for sustainable indoor farming, where optimal growth is achieved through efficient resource management rather than maximum resource input.

### 6. SUMMARY

This dissertation presents the development of an IoT-enabled indoor farming system integrated with mathematical and machine-learning models for optimizing plant growth in Controlled Environment Agriculture (CEA). Lettuce (*Lactuca sativa* L.) was used as the model crop under controlled environmental conditions.

A four-dimensional logistic growth model was developed to describe plant height, biomass, chlorophyll content, and leaf area simultaneously. An allometric model linking plant height and biomass achieved high predictive accuracy ( $R^2 = 0.9388$ ), while a Relative Growth Rate (RGR) model demonstrated strong performance across experimental treatments.

The study introduced novel metrics, including the Growth Efficiency Ratio (GER) and a machine-learning-based Plant Growth Index (PGI), for evaluating resource-use efficiency and plant performance. Models relating Leaf Area Index (LAI) to light-use efficiency were also developed to identify optimal cultivation conditions.

The implementation of IoT-based adaptive control significantly improved biomass production, plant growth, resource-use efficiency, and system responsiveness. The results demonstrate that combining IoT technologies, machine learning, and advanced growth models provides an effective framework for sustainable and resource-efficient indoor farming.

## 7. APPENDICES

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